PVASS:

- A non-terminal B is non-linear if $B \xrightarrow{*} \cdots B \cdots B \cdots$
- Goal: Approx SCCs from bottom to up.
- $\mathcal{C}$-PVASS: a GVASS with terminals being relations in $\mathcal{C} \cap (\mathbb{N}^d \times \mathbb{N}^d)$ that are monotone, where $\mathcal{C}$ is effectively approximable.

Thm: Sections of $\mathcal{C}$-PVASS is effectively approximable

Note: W.l.o.g we can assume the rule-graph is strongly connected. and $\exists$ non-linear non-terminal. (Induction)

Note: If $\mathcal{C}$ is monotone then $\mathcal{C}$-PVASS sections are monotone.

- $\mathbb{Z}$-VAS simplifier of $\mathcal{C}$-PVASS V:

① Assume w.l.o.g., each terminal $a \in \Sigma$ has $a \trianglelefteq b(a) + \mathbb{N}(F(a))$.

② Create a normal PVAS: $S \to \Delta(f)\, S$ for each $f \in F(a)$

$S \to \Delta(b(a))$.

③ For a normal PVAS w/ d counters and n nonterminals.

(Esparza '94) Create a $(d+n)$-VAS:

for a rule $A \to \vec{v}_1 A \vec{v}_2 B \vec{v}_3 BC \cdots$

add a rule $(\vec{v}_1 + \vec{v}_2 + \cdots, \underset{A}{(1-1)}, \underset{B}{2}, \underset{C}{1})$

$\mathbb{Z}$-simplify (V)

This is a BPP.

$\mathbb{Z}$-Reach $= \mathbb{N}$-Reach ← use ILP to solve how often a rule is taken

The characterization ILP will be defined naturally for the $\mathbb{Z}$-simplifier.

- Thm: Let V be a strongly connected, non-linear e-GVAS

where every non-terminal is $\trianglelefteq b(a) + \mathbb{N}(F(a))$.

Assume:

(1) [Pumpable]: $\exists$ rule $S \to w S w'$ s.t.

$$x \xrightarrow{w} > x \qquad > y \xrightarrow{w'} y$$

(2) In above ILP, every $\underbrace{\text{variable}}_{\text{one for each period?}}$ unbounded

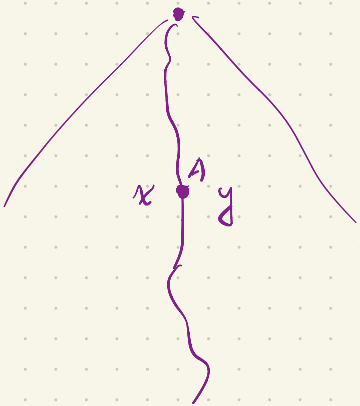
Then: $x \xrightarrow{S} y$.

Note: need to define $\mathbb{Z}$-runs of PVASS as;

any run from $x+m$ to $y+m$ for $m \geq 0$

is a $\mathbb{Z}$-run from x to y.

- Unpumpable case: on the derivation tree, $\exists$ one branch



$\exists$ dimension $i \in [d]$,

either $\begin{cases} \forall\, (x\,A\,y) \in \text{this branch}, & x[i] \leq B \\ \forall\, (x\,A\,y) \text{ — — — — —} & y[i] \leq B. \end{cases}$

for some B very hard to compute.